Collateralized Commodity Obligations: Valuation and Rating

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Abstract

In this article we address valuation and rating of Collateralized Commodity Obligations (CCO), which are relatively new structured products similar to the Collateralized Debt Obligation (CDO). Commodities as an asset class have been in the spotlight of investors’ attention for the last decade, and CCOs, which have the form of fixed income instruments, give fixed income investors an exposure to commodity markets. Underlying assets of a CCO are Commodity Trigger Swaps (CTS), similar to a Credit Default Swap, but instead of a default, a “credit event”, or a “trigger” occurs when a commodity price reaches a certain pre-set level.

Rating agencies have used their CDO evaluators to rate CCOs; however, particular characteristics of commodity prices and an abundance of historical price data for commodities render such an approach questionable. Recently, S&P has withdrawn their ratings for CCOs, suggesting problems with their approach to rating of these structured products.

We examine the historical performance of CCOs and suggest two approaches to valuating and rating them. The first is a flexible multivariate parametric model for commodity prices: a mean-reversion with correlated trends. The second approach is close in spirit to the historical simulation method for risk management and is based on the block bootstrap technique. We apply both approaches to an example of a CCO and compare the results to the ratings provided by the rating agencies. We find that:

- Simulated ratings are quite sensitive to the model assumptions.
- The default probabilities resulting from the agencies’ ratings underestimate both historically observed and bootstrap-simulated default probabilities;
- The non-parametric approach most closely matches the historically observed probabilities of default.

The results demonstrate the benefit of the data-driven, non-parametric modelling approach to rating CCOs.
Structure of a typical CCO

In December 2004, Barclays Capital launched the world’s first Collateralized Commodity Obligation (CCO). The CCO, having a form of a fixed income instrument (a note), was developed to provide fixed income investors access to commodities as an asset class. Commodities as an asset class have been in the spotlight of investors’ attention for the last decade, which is confirmed by exponentially growing trading volumes in commodity futures and emergence of various commodity investment vehicles such as Exchange-Traded Funds (ETFs).

Previously, any commodity investment opportunity (futures, indices) yielded equity-like returns. CCOs transform commodity price exposure to a ratable product and provide investors with a bond-like return that is linked to the performance of an underlying portfolio of commodities. The main advantage of adding commodity exposure to a fixed income portfolio is diversification, as the commodity returns historically had low correlations to interest rates and credit spreads. The biggest investors in CCOs, according to Schwab [2005], are insurance companies, commercial banks and hedge funds in Europe, the USA and Asia.

The CCOs are structured similarly to the well-known Collateralized Debt Obligations (CDOs). The CDOs and CCOs are both, at a simple level, transactions that transfer risk of a reference portfolio of assets. For the CDO it concerns a transfer of credit risk, for the CCO it concerns a transfer of commodity price risk.

Recall that a Collateralized Debt Obligation (CDO) is created from a basket of underlying bonds, loans or credit default swaps. The underlying securities in a CCO comprise, for example, 100 derivatives known as commodity trigger swaps (CTS), based on up to 16 precious metal, base metal, agricultural and energy prices. The structure of a commodity trigger swap is similar to that of a credit default swap (CDS). There are two main differences: first, the reference entity of CTS is a certain commodity price (rather than a company that might default); and second, the credit event of CDS is replaced by the so-called trigger event, which occurs when the corresponding commodity price falls under (or rises above) some pre-set level at the time $T$, which is the contract’s maturity date. Note that, in the case of CTS, the protection buyer pays premia until $T$ (in the case of CDS, the payments are made either until maturity or the default of a reference entity, whichever comes first).

Another difference between CDS and CTS is that the recovery rate of CTS is 0%, which implies 100% loss given default. A CTS can be seen as a European binary option (usually deep in- or out-of-money).

![Diagram of a CCO structure](image)
Exhibit 1. Basic structure of a CTS. C is the reference commodity, firm A is the protection buyer and firm B is the protection seller. $S_T(C)$ is the price of commodity C at time T, x is the level of the trigger event.

The asset side of a CCO is formed by a portfolio of e.g., 100 commodity trigger swaps (CTS), based on up to 16 commodity prices. This portfolio is sold to a special-purpose vehicle (SPV). Just as in a CDO, the liability side of the SPV is formed by issuing notes belonging to tranches of increasing seniority. Losses encountered by CTSs will first affect the 'equity’ tranche, then the ‘mezzanine’ tranches, and finally the ’senior’ tranches. The bond-style payoff of the CCO tranches consists of periodic interest payments of a floating rate (e.g., LIBOR) plus an additional premium, depending on the tranche. Besides the periodic interest payments, at the time of maturity the CCO also pays the principal less the losses generated by trigger events.

The cumulative loss of the portfolio, as well as the notional of a CCO tranche depends on the total number of trigger events of the underlying CTSs that occur at the time of maturity $T$. Let $L$ be the total number of trigger events in the CCO portfolio at maturity. Consider a CCO with $k$ tranches indexed by $\kappa \in \{1, \ldots, k\}$ and attachment points $0 = K_0 < K_1 < \cdots < K_k$. The notional of a tranche $\kappa$ at maturity depends exclusively on the total number of trigger events at maturity $L$ and equals to

$$N_k(T) = N_k(0)f_k(L)$$

with $f_k(L) = \begin{cases} 
1, & \text{if } L < K_{k-1}, \\
\frac{K_k - L}{K_k - K_{k-1}}, & \text{if } L \in [K_{k-1}, K_k], \\
0, & \text{if } L > K_k.
\end{cases}$

In contrast to CDOs, where losses are generated by firms’ defaults (so investors are exposed to credit risk), in CCOs, losses are due to movements in the underlying commodity prices (hence, the exposure to market risk). One very important difference between CDOs and CCOs, especially relevant in view of the recent financial crisis, is a complete transparency of the CCO structure. While in a typical loan-based (e.g., mortgage-based) CDO it is unknown which assets comprise a given CDO (who are the loan holders, what is their credit history and so on), it is absolutely clear what kinds of (standardized) commodities underlie a given CCO. Also, contrary to assets underlying a CDO, where there might be no historical data on these assets’ performance, there is an abundance of historical data of commodity prices. We shall exploit this fact by means of a data-driven historical simulation method for assessing CCO risks and performance. First we discuss issues arising in CCO valuation and rating and describe methods employed by the rating agencies.

**CCO valuation and rating**

While there is an abundant literature on pricing and rating of CDOs (see, for example, Hull and White [2004, 2006, 2008]), there is practically nothing in the academic literature about the same issues for CCOs; the only exception is the paper by Chang et al. [2009], where structured products combining credit and commodity price risk (CDCO) are considered. Standard & Poor’s (S&P) and Fitch rating agencies have the descriptions of CCOs and their rating methodologies on their websites (see Standard
Poor’s [2006], Fitch [2006]), since the issued commodity-linked notes (i.e., CCO tranches) were rated by them. Long before that, these agencies had developed rating methodologies for credit-linked products (CDOs). When rating CCOs, the agencies heavily relied on their CDO methodologies, simply using their trusted CDO evaluators with minor modifications. As of April 2010, the S&P has withdrawn their ratings for CCO. This might be considered as a failure of its rating approach and definitely indicates the need for alternative methods towards CCO risk assessment.

S&P used standard uncorrelated mean-reversion models for commodity prices. The probabilities of trigger events were then determined analytically. Subsequently, these probabilities (which replaced the default probabilities) were combined using the standard CDO evaluator, based on the Gaussian copula model (Standard & Poor’s [2005]). The mean-reverting processes for commodities were assumed to be uncorrelated but the trigger events were made correlated by using the historical returns correlations between commodities in the Gaussian copula. Similarly, Fitch used its CDO evaluator (Fitch [2004]) with the input being the probabilities of trigger events, determined by Monte Carlo simulations. Fitch used the factor commodity price model, which combined the Principal Component Analysis, a nonlinear GARCH process, a constant mean and jumps. In this approach, the correlations between commodities were taken into account already at the first step. Both approaches, although quite different, seem rather ad-hoc and were obviously designed with the existing CDO evaluators in mind.

The rating of a structured note is an indicator (often not very reliable) for the implied risk. As the recent crisis has shown, investment decisions cannot be based exclusively on ratings, since this often implies high model risk. Price models used by the rating agencies are rigid parametric models, undoubtedly describing well some features of commodity prices. Under certain market conditions, parametric models can be very efficient, but also can become very unreliable when major changes in the prices’ behavior (non-stationarities) take place. We will demonstrate this when examining CCO evaluation with a parametric model.

The main challenges in developing a CCO rating methodology are two-fold:

- Devising a realistic model for each commodity price evolution (which persists for at least the next 5 years, which is the maturity of a typical CCO).
- Modeling the inter-dependencies (by means of e.g., returns’ correlations) between prices of commodities in the reference portfolio.

Both issues are equally important and should be addressed simultaneously, by developing a realistic multivariate commodity price model.

Commodity prices are often modeled by means of a mean-reversion process. Price developments in oil and metals fuel the ongoing discussion whether mean-reversion is still a valid assumption for energy and other commodities (see e.g., Geman [2005]). For example, the recent developments in the palladium and gold prices reject the assumption that they can be modeled by a mean-reversion model, since these prices have risen considerably for a long period. Other commodity prices such as those of lead and copper may well be modeled by a mean-reversion, but with a higher long-term mean than in the previous decade. In all, under the current changing market conditions, it is questionable whether any parametric model, no matter how realistic, will continue to describe well the commodity prices for the next e.g., 5 years.
Although the discussion which model is the correct one may be slightly philosophical, we show that the model choice has a huge impact on the CCO rating results. In contrast to credit risk events (defaults of loans or CDS’s), there is plenty of historical data on commodity prices, which can be explored for CCO risk assessment. Therefore, we suggest using a model-free, non-parametric approach, based purely on historical data: block bootstrap simulation. The essence of this method is resampling blocks of historical commodity returns, *simultaneously for all commodities in the portfolio*, which preserves their inter-dependencies. In this method, we can use a long history of commodity returns, which are amply available (for example, in our simulation study we shall use 17 years of data). This ensures we resample from a wide variety of economic environments represented by the data and makes sure we get a wide range of simulated price movements. Next we examine the historical performance of a typical CCO example.

**Historical performance: the Goldman Sachs CCO**

In this section we investigate the historical performance of a CCO issued by Goldman Sachs International (GSI) in 2006. The asset side of the CCO consists of the portfolio of 100 CTSs in 12 different commodities with maturity of 5 years. For each commodity, upper and lower trigger event strikes are defined as percentages of the corresponding commodity price at the time of issue. The trigger event strikes are evenly distributed between the upper and the lower strike. Exhibit 2 shows the lower and upper strike for each commodity, as well as the distribution of CTSs among the different commodities. Note that trigger events occur if commodity prices fall, as all strikes are below 100%. (There are examples of CCO where trigger events occur when commodity prices rise, i.e., with strikes above 100%.)

<table>
<thead>
<tr>
<th>No.</th>
<th>Commodity</th>
<th>Upper Trigger Event Strike</th>
<th>Lower Trigger Event Strike</th>
<th># of Trigger Events (CTSs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Aluminum</td>
<td>43%</td>
<td>35%</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>Tin</td>
<td>49%</td>
<td>42%</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>Zinc</td>
<td>25%</td>
<td>17%</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>Nickel</td>
<td>22%</td>
<td>14%</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>Silver</td>
<td>31%</td>
<td>24%</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>Platinum</td>
<td>30%</td>
<td>23%</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>Palladium</td>
<td>34%</td>
<td>27%</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>Corn</td>
<td>80%</td>
<td>74%</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>Wheat</td>
<td>59%</td>
<td>51%</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>Sugar</td>
<td>79%</td>
<td>72%</td>
<td>8</td>
</tr>
<tr>
<td>11</td>
<td>Copper</td>
<td>20%</td>
<td>12%</td>
<td>9</td>
</tr>
<tr>
<td>12</td>
<td>Lead</td>
<td>35%</td>
<td>28%</td>
<td>8</td>
</tr>
</tbody>
</table>

**Exhibit 2.** Asset side of the GSI CCO: underlying commodities, upper and lower trigger strikes and number of CTS’s per commodity.

The liability side of the CCO consists of notes belonging to 5 tranches of increasing seniority: BBB, A, AA, AAA, and Super Senior. Each tranche has attachment and detachment points (Exhibit 3). If the total number of trigger events at maturity is lower than the attachment point, then the notional of the corresponding tranche remains intact; if it is between the attachment and detachment point, then part of the notional is lost; and if the number of trigger events is bigger than the detachment point then the whole notional is lost.
Exhibit 3. Attachment and detachment points for the tranches of the GSI CCO.

For example, suppose that in 5 years time, the prices of lead, copper and wheat fall to, respectively, 30%, 13% and 55% of their value at time zero, while the prices of all other commodities stay above their upper strikes. In such a scenario, the total number of trigger events is 19. This will result in a loss of 100% of the notional of the BBB tranche and 33.3% of the notional of the A tranche.

To investigate the historical performance of this CCO, the historical daily closing commodity prices from July 1st 1993 until January 31st 2010 were obtained from Thomson Datastream. Exhibit 4 presents the list of commodities, their grade and quality specifications used in the GSI CCO example.

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Code</th>
<th>Type/Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>LAHCASH</td>
<td>LME-Aluminium 99.7% Cash $/MT - A.M. OFFICIAL</td>
</tr>
<tr>
<td>Copper</td>
<td>LCPCASH</td>
<td>LME-Copper, Grade A Cash $/MT - A.M. OFFICIAL</td>
</tr>
<tr>
<td>Lead</td>
<td>LEDCASH</td>
<td>LME-Lead Cash $/MT - A.M. OFFICIAL</td>
</tr>
<tr>
<td>Tin</td>
<td>LTICASH</td>
<td>LME-Tin 99.85% Cash $/MT - A.M. OFFICIAL</td>
</tr>
<tr>
<td>Zinc</td>
<td>LZZCASH</td>
<td>LME-SHG Zinc 99.995% Cash $/MT - A.M. OFFICIAL</td>
</tr>
<tr>
<td>Nickel</td>
<td>LNICASH</td>
<td>LME-Nickel Cash $/MT - A.M. OFFICIAL</td>
</tr>
<tr>
<td>Silver</td>
<td>SLVCASH</td>
<td>Silver Fix LBM Cash dollarcents/Troy ounce</td>
</tr>
<tr>
<td>Platinum</td>
<td>PLATFRE</td>
<td>London Platinum Free Market $/Troy ounce</td>
</tr>
<tr>
<td>Palladium</td>
<td>PALLADM</td>
<td>Palladium $/Troy ounce</td>
</tr>
<tr>
<td>Corn</td>
<td>CORNUS2</td>
<td>Corn No.2 Yellow dollarcents/Bushel</td>
</tr>
<tr>
<td>Wheat</td>
<td>WHEATSF</td>
<td>Wheat No.2, Soft Red dollarcents/Bushel</td>
</tr>
<tr>
<td>Sugar</td>
<td>WSUGDLY</td>
<td>Raw Sugar-ISO Daily Price dollarcents/pound</td>
</tr>
</tbody>
</table>

Exhibit 4. Technical description of the commodities.

The historical performance analysis is based on a hypothetical daily CCO issue from July 1, 1993, up to January 31, 2005. The maturity of the contract is 5 years. The notional of tranches at maturity depends on the total number of trigger events generated by the portfolio of CTSs. For each hypothetical CCO, we calculate the number of trigger events and the realized present value of all payments for every CCO tranche.

Until 2003, most of the commodity prices underlying the CCO fluctuated around their long-term means. From 2003 on, the prices gradually started to show a strong upward trend and this trend persisted until the first quarter of 2008. (In Appendix, graphs of historical commodity prices are shown). Thus, no trigger events occurred after 2002. However, by the end of 2008, all commodity prices except for sugar dropped
significantly, but the CCOs issued after 2004 (for which the period from issue to maturity includes the crisis period) still experienced no trigger events. This can be explained by the fact that, despite the major drop in commodity markets, the prices decreased to levels higher than the average historical levels up to 2003.

The total number of trigger events is calculated for the CCO issued every day from July 1, 1993 until 31\textsuperscript{st} January 2005 and shown in Exhibit 5.

Exhibit 5. The number of trigger events and the attachment points of the tranches.

The historical analysis shows that the investors in the BBB tranche lose some of their notional in 16.8% of the hypothetically issued CCOs. The investors in the A and AA tranches lose some of their notional in, respectively, 2.49% and 1.42% of the issued CCO. The investors in AAA and Super Senior tranches experience no losses.

We also calculated the realized present value for each tranche \( i \) of each hypothetical issue of the CCO as:

\[ PV_i = \text{Disc. bond payments} + \text{Disc. principal} - \text{Disc. Losses}. \]

The descriptive statistics for the whole historical period are shown in Exhibit 6. Exhibit 7 shows the realized present value of tranches, and a pattern very similar to that in Exhibit 5 can be seen. Before mid-1997, the present value of the tranches was quite volatile. After that, due to a very low number of trigger events, the tranche values remain almost constant.

<table>
<thead>
<tr>
<th>1993-2005</th>
<th>BBB</th>
<th>A</th>
<th>AA</th>
<th>AAA</th>
<th>SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.07312</td>
<td>1.091157</td>
<td>1.077581</td>
<td>1.050065</td>
<td>1.026809</td>
</tr>
<tr>
<td>Median</td>
<td>1.148202</td>
<td>1.106539</td>
<td>1.081342</td>
<td>1.049822</td>
<td>1.026744</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.186831</td>
<td>0.103708</td>
<td>0.037065</td>
<td>0.000913</td>
<td>0.000211</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.373167</td>
<td>0.332290</td>
<td>0.694311</td>
<td>1.048591</td>
<td>1.026348</td>
</tr>
<tr>
<td>Maximum</td>
<td>1.159459</td>
<td>1.113969</td>
<td>1.086675</td>
<td>1.052557</td>
<td>1.027537</td>
</tr>
</tbody>
</table>


The statistical summary of the realized present values shows that, from a historical perspective, the BBB and A tranches are the least desirable, combining a lower

\[ 1 \text{ We used the daily five year UK swap rate for coupons and discounting.} \]
average present value with a higher volatility. Note that, because triggers can only be hit after 5 years, the discounted interest rate payments result into the minimum value which is well above 0.

After mid-1997, the CCO would perform significantly better: all tranches’ returns outperform the risk-free rate of return (e.g., 5-year UK swap rate). The descriptive statistics for the present value of the CCOs issued from 1998 until 2005 (Exhibit 8) show significantly higher average realized present values and lower risk for tranches BBB, A and AA.

Based on this analysis, we can conclude that all tranches would perform well (especially in the period of rising commodity prices (1998-2008)) and even the most risk taking investors (BBB-investors) would have made a profit.

![Graph showing realized present values of tranches at a given issue date.](image)

**Exhibit 7.** Realized present values of tranches at a given issue date.

<table>
<thead>
<tr>
<th>1998-2005</th>
<th>BBB</th>
<th>A</th>
<th>AA</th>
<th>AAA</th>
<th>SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.152011</td>
<td>1.109049</td>
<td>1.083272</td>
<td>1.05105</td>
<td>1.02742</td>
</tr>
<tr>
<td>Median</td>
<td>1.151286</td>
<td>1.108375</td>
<td>1.082634</td>
<td>1.050455</td>
<td>1.026853</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.003667</td>
<td>0.002474</td>
<td>0.001759</td>
<td>0.000866</td>
<td>0.000225</td>
</tr>
<tr>
<td>Minimum</td>
<td>1.144216</td>
<td>1.103634</td>
<td>1.079284</td>
<td>1.048846</td>
<td>1.026367</td>
</tr>
<tr>
<td>Maximum</td>
<td>1.159459</td>
<td>1.113969</td>
<td>1.086675</td>
<td>1.052557</td>
<td>1.027537</td>
</tr>
</tbody>
</table>

**Exhibit 8.** Descriptive statistics for CCO tranches’ realized present value (issue 1998-2005).

All these results are, however, completely backward-looking, so such a historical performance analysis can hardly be used to assess future risks associated with CCOs. So in the next two sections we assess several parametric models for rating and valuing CCOs and describe a new model-free historical simulation approach to this problem.
Valuation methods

A parametric model: mean-reversion with trend

Both Fitch and S&P use parametric models for commodity prices (standard uncorrelated mean reversion in case of S&P, factor model with NGARCH volatility and jumps in case of Fitch). However, these models are used only for calculating (analytically or by means of simulations) the probabilities of trigger events, which are then inserted into CDO evaluators. We find this approach questionable; moreover, neither rating agency is particularly clear about their exact methodology. So to assess the performance and robustness of parametric models, we will directly model an ensemble of commodity prices by means of correlated mean reversions with non-constant means. This multivariate model is flexible and arguably more realistic than those used by the rating agencies. It directly incorporates the dependencies between the commodities by means of the covariance matrix of the returns. Moreover, it combines the mean-reversion as a standard model for commodity prices, while allowing for long-term trends such as those recently observed in many commodity markets.

The mean reversion model is the most frequently used model for commodity prices (see e.g., Schwartz [1997], Geman [2006]). The logarithm of the commodity price $X_t = \ln S_t$ is modeled as an Ornstein-Uhlenbeck process, given by the following stochastic differential equation:

$$dX_t = \kappa(\alpha - X_t)dt + \sigma dW_t,$$

where $\kappa$ is the mean reversion rate, $\alpha$ is the long run mean, $\sigma$ is the volatility and $W_t$ is the standard Brownian motion. For energy and other commodity prices, there is an ongoing discussion whether prices still follow mean reversion (Geman [2005]). For many commodities (energy, metals), prices show large upward trends since 2003, often attributed to a growing demand from China (for illustration of these price trends, see graphs in Appendix). Mean reversion with a constant mean obviously cannot cope with such situations. An alternative would be to adjust the equation (1) by allowing the mean value $\alpha$ to vary over time.

Recall that, for the purposes of CCO valuation, we need to model several correlated commodity prices simultaneously. Generalizing (1) to multiple commodities, we have

$$dX_{it} = \kappa_i(\alpha_{it} - X_{it})dt + \sum_{j=1}^{g} c_{ij} dW_{jt},$$

where $c_{ij}$ are the elements of the Cholesky decomposition of the variance-covariance matrix $\Sigma$ and $W_{1t}, ..., W_{gt}$ are $g$ independent Brownian motions. To get around the unrealistic assumption of constant mean price, we can use the so-called mean reversion with trend.

We assume that the long-run mean log-price $\alpha_{it}$ is not constant but represents a so-called trend, which is slowly (and stochastically) varying over time. The trend is filtered out from the historical price series by a state-space method using the Kalman filter. We used the so-called local linear trend model, or a smooth trend model, by
For a univariate time series $y_t$, it is the model of the form

$$y_t = \mu_t + \beta_t + \omega_t, \quad \omega_t \sim N(0, \sigma^2_\omega),$$
$$\mu_{t+1} = \mu_t + \beta_t,$$
$$\beta_{t+1} = \beta_t + \eta_t, \quad \eta_t \sim N(0, q \sigma^2_\eta),$$

where $\mu_t$ is the level and $\beta_t$ is the slope. The addition of normally distributed random variables with non-zero variance allows the trend level and slope to vary over time. The parameter $q$ is the signal-to-noise ratio. It determines how sensitive the estimated trend is to short-term fluctuations in the time series. The lower the value of $q$, the less sensitive the estimated trend is to these fluctuations. We illustrate this approach on the example of the price of aluminium in the last 17 years. Exhibit 9 shows the aluminium log-price, together with trends filtered out using the signal-to-noise ratios $q$ of $3 \cdot 10^{-9}$, $6 \cdot 10^{-9}$ and $9 \cdot 10^{-9}$. Note that, for a lowest signal-to-noise ratio, the estimated trend does not pick up the recent recovery of aluminium price, while for the highest (and medium) value of $q$ this recovery is clearly visible. Generally, the choice of $q$ is quite arbitrary and a matter left to the model builder. However, in the next section we show that this choice is quite crucial when forecasting the price trends with the local linear trend model.

The smooth trend model can be easily generalized to the case of a multivariate time series. In this case, it is important to realize that the trends for different commodity prices are highly correlated. These trend correlations are summarized in the matrix $R$ which can be estimated from the filtered out trends.

For the purpose of simulations, the trend is forecasted until the CCO’s maturity $T$ (for details on how to forecast trends, we refer to Durbin and Koopman [2001]). Let the mean forecasted trends for all commodities at time $T$ (minus its value at time $t=0$) be...
summarized in the vector $m$, and let the standard errors of the forecasts will be in the diagonal matrix $D$. The (estimated) covariance matrix $\Omega$ of the future trends is
\[ \Omega = D R D. \]  
(3)
Let $B$ be the Cholesky decomposition of $\Omega$. Then a trend value at a future date $T$ (again, relative to its value at time zero) can be simulated as a normal random vector with mean $m$ and variance-covariance matrix $\Omega$, by taking:
\[ r = m + B Z, \]  
(4)
where $Z$ is a drawing from the multivariate standard normal distribution.

Having filtered out the trends from all commodity prices, we can set $\alpha_{\nu} = 0$ in (2), obtaining the zero-mean Ornstein-Uhlenbeck process for the detrended log price $\tilde{X}_t$:
\[ d\tilde{X}_t = -\kappa t \tilde{X}_t dt + \sum_{j=1}^{\infty} c_{ij} dW_{ji}. \]  
(5)
The mean reversion speeds $\kappa$ (and the variance-covariance matrix $\Sigma$) are estimated by discretizing (5) by e.g., Euler scheme and using a computationally efficient way to compute the Generalized Least Squares estimator of $\kappa_i$’s and $\Sigma$, given by Davidson and MacKinnon [2004].

The next step in the simulation procedure is generating drawings from the zero-mean mean reversion model (5), at time $T$. Recall that the solution to (5) is given by
\[ \tilde{X}_t = e^{-\kappa T} \tilde{X}_{i0} + e^{-\kappa T} \int_0^T e^{\kappa s} \sum_{j=1}^{\infty} c_{ij} dW_{ji}. \]  
(6)
From (5) it follows that the distribution of $\tilde{X}_{ij} - \tilde{X}_{i0}$ is normal with
\[
E[\tilde{X}_{ij} - \tilde{X}_{i0}] = (e^{-\kappa T} - 1) \tilde{X}_{i0},
\]
\[
Var[\tilde{X}_{ij} - \tilde{X}_{i0}] = e^{-2\kappa T} \int_0^T e^{2\kappa s} \sigma_i^2 ds = \frac{\sigma_i^2}{2\kappa_i} (1 - e^{-2\kappa T}),
\]  
(7)
\[
Cov[\tilde{X}_{ij} - \tilde{X}_{i0}, \tilde{X}_{ij} - \tilde{X}_{j0}] = \frac{\sigma_{ij}^2}{\kappa_i + \kappa_j} (1 - e^{-T(\kappa_i + \kappa_j)}),
\]
where $\sigma_{ij}^2$’s are the elements of the variance-covariance matrix $\Sigma$.

The simulated commodity log-prices at CCO’s maturity $T$ are obtained by adding the simulated trends given in (4) to the simulated mean reversion values whose distributional characteristics are given in (7). Subsequently, the total number of trigger events is calculated for each simulation. The loss distribution of CCO tranches is obtained by the Monte Carlo simulation procedure, repeating the above steps many times.

One simplification of the above model is reducing the number of fundamental factors (sources of uncertainty) driving commodity prices. In the model (2) there are as many sources of uncertainty as there are commodities in the reference portfolio. However, due to correlations between commodity prices, it is possible to reduce the number of these factors by means of e.g., Principal Component Analysis (see for example, Smith [2002]), as this is done in the Fitch [2006] model.
The above multivariate model for commodity prices seems quite flexible: it partially deals with non-stationarities (in mean) of the price series, while modelling dependencies between commodity prices trends (long-term fluctuations) and daily returns (short-term fluctuations). In the next section we test the performance of this model (on the basis of simulations) for CCO ratings and show that its performance is not very robust to the choice of the signal-to-noise ratio. The alternative is offered by a non-parametric simulation approach: the moving block bootstrap.

**Moving block bootstrap approach**

Bootstrapping is a statistical technique, introduced by Efron [1979] for assessing properties of estimators, such as variance or sampling distribution, by resampling the data from an empirical distribution. In practice, it amounts to repeatedly drawing (with replacement) the data from the original sample and re-calculating the quantity of interest on the basis of a new data sample. For this method to work well, the data must represent independent drawings from a distribution. If there is a serial dependence in the data (such as in case of a time series), the resampling would destroy this dependence. The solution was provided by Carlstein [1986] and Kunsch [1989] (and further improved by Lall and Sharma [1996]), who suggested to resample not the individual observations, but blocks of them (hence the name “moving block bootstrap”). The serial dependence within blocks is preserved in this way, but the dependence between blocks is destroyed as those are resampled independently. However, if blocks are chosen long enough and the serial dependence in the data series is not too strong, arguably this has a minimal effect.

More formally, the moving block bootstrap amounts to randomly selecting blocks of consecutive observations with replacement from a set of overlapping blocks. Given the data series $X = \{X_i, 1 \leq i \leq n\}$ from a (possibly multivariate) stationary time series, the method constructs a set of blocks $B_1, B_2, ... , B_b$ of length $l$, where $B_i = \{X_{i1}, ..., X_{il}\}$, $b = n - l + 1$ and $X_{ij} = X_{i+j-1}$. Blocks are drawn randomly (and independently) from the set $B_1, B_2, ... , B_b$ and joined together to form a simulated time series.

Here we shall use the moving block bootstrap for approximating the loss distribution of CCO tranches. The moving block bootstrap is applied to the multivariate series of commodity returns. It preserves the cross-correlation structure of the commodities by selecting blocks of returns for different commodities with equal starting (and ending) date. As mentioned before, the drawback of the block bootstrap is the bias for strongly dependent data. One of the stylized facts of financial returns is that they are (nearly) uncorrelated (see e.g., McNeal et al. [2005]), so this drawback is of little concern when applied to CCO returns. The absolute or squared returns are, however, correlated, indicating volatility clustering. This feature is of no significant importance here, since the CCO performance depends exclusively on the commodity prices at time of issue and maturity. Nevertheless, this autocorrelation structure is partly preserved by the moving block bootstrap. The choice of the block length is the matter left to the model builder; however, in the simulation study we show that the method is robust to the block length.
Simulation study

Mean-reversion with trend

We run an extensive simulation study to assess the ratings of CCO tranches for the CCO studied above (Goldman Sachs CCO with a 5-year maturity). We compare the results with the ratings given by the rating agencies. For this, we need the probabilities of default as defined by the agencies’ ratings. Exhibit 10 shows the probabilities of default for different ratings by S&P and Fitch. The agencies define the “default” of a tranche as an event when some part of the notional is lost at maturity $T$. However, in our simulation study, we simulate probabilities for two types of events: the entire notional of a tranche is lost ($N_T = 0$) and a part of the notional is lost ($0 \leq N_T < N_0$).

<table>
<thead>
<tr>
<th>Credit rating</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(default) in % by S&amp;P</td>
<td>0.118</td>
<td>0.356</td>
<td>0.709</td>
<td>2.812</td>
</tr>
<tr>
<td>P(default) in % by Fitch</td>
<td>0.030</td>
<td>0.200</td>
<td>0.560</td>
<td>1.580</td>
</tr>
</tbody>
</table>

Exhibit 10. Credit curve for tranches, by S&P and Fitch for a CDO with 5 years to maturity.

Exhibits 11 to 13 show the results (probabilities of default) for 500 000 simulations of the mean reversion with trend, with signal-to-noise ratios $q$ of $2 \cdot 10^{-9}$, $3 \cdot 10^{-9}$ and $5 \cdot 10^{-9}$. The probabilities of default are given in percent; the standard errors of simulated values are in parentheses.

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Super Senior</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(N_T = 0)$</td>
<td>0</td>
<td>0.01</td>
<td>0.21</td>
<td>1.14</td>
<td>2.50</td>
</tr>
<tr>
<td>(0)</td>
<td>(0.002)</td>
<td>(0.007)</td>
<td>(0.015)</td>
<td>(0.022)</td>
<td></td>
</tr>
<tr>
<td>$P(0 \leq N_T &lt; N_0)$</td>
<td>0.01</td>
<td>0.16</td>
<td>0.89</td>
<td>1.93</td>
<td>99.45</td>
</tr>
<tr>
<td>(0.001)</td>
<td>(0.006)</td>
<td>(0.013)</td>
<td>(0.020)</td>
<td>(0.010)</td>
<td></td>
</tr>
</tbody>
</table>

Exhibit 11. Simulated probabilities of default for CCO tranches (mean reversion with trend, $q=2 \cdot 10^{-9}$).

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Super Senior</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(N_T = 0)$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.18</td>
<td>0.41</td>
</tr>
<tr>
<td>(0)</td>
<td>(0)</td>
<td>(0.001)</td>
<td>(0.006)</td>
<td>(0.009)</td>
<td></td>
</tr>
<tr>
<td>$P(0 \leq N_T &lt; N_0)$</td>
<td>0</td>
<td>0.01</td>
<td>0.14</td>
<td>0.32</td>
<td>99.35</td>
</tr>
<tr>
<td>(0)</td>
<td>(0.001)</td>
<td>(0.005)</td>
<td>(0.008)</td>
<td>(0.011)</td>
<td></td>
</tr>
</tbody>
</table>

Exhibit 12. Simulated probabilities of default for CCO tranches (mean reversion with trend, $q=3 \cdot 10^{-9}$).

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Super Senior</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(N_T = 0)$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.006</td>
<td>0.018</td>
</tr>
<tr>
<td>(0)</td>
<td>(0)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.007)</td>
<td></td>
</tr>
<tr>
<td>$P(0 \leq N_T &lt; N_0)$</td>
<td>0</td>
<td>0</td>
<td>0.005</td>
<td>0.014</td>
<td>97.77</td>
</tr>
<tr>
<td>(0)</td>
<td>(0)</td>
<td>(0.001)</td>
<td>(0.005)</td>
<td>(0.010)</td>
<td></td>
</tr>
</tbody>
</table>

Exhibit 13. Simulated probabilities of default for CCO tranches (mean reversion with trend, $q=5 \cdot 10^{-9}$).
As the above tables show, the results largely depend on the signal-to-noise ratio $q$. Recall that a relatively large value of $q$ gives a lot of weight to the latest observations in the sample period so that the simulated trend will be also largely influenced by the most recent observations.

Many commodities experienced large price movements over the last few years. First there was a huge increase in commodity prices, which ended in mid-2008, when a sharp decline set in. In the period mid-2009 – mid-2010, many commodity prices have recovered. This rollercoaster makes modelling commodity prices with a mean reversion model rather hard, because it is difficult to determine the right long-term mean. The model with the stochastic trend picks up some of the recent recovery for the higher signal-to-noise ratio, but not for all commodities.

For the lowest signal-to-noise ratio ($q=2\cdot10^{-9}$), the recovery has not set in the stochastic mean, ensuring many triggers (for different commodities) are hit, resulting into rather large default probabilities. For higher values of $q$, the recovery of many commodity prices is picked up in the stochastic trend. This means that for these values of $q$, the higher graded tranches have quite low default probabilities. The only difference is the BBB tranche, where the results for all values of $q$ show that investors almost certainly lose part (but not all) of their notional. On closer inspection, this effect is caused by an exceptionally large decline in corn and wheat prices over the period mid-2008 – mid-2009. In the composition of our example CCO, corn has high trigger event levels (74% - 80% of the starting value). This means that even for a slightly decreasing mean trend (shown in Exhibit 14), these trigger levels will be definitely hit 5 years from now, which implies a very high probability that the BBB tranche is hit.

Exhibit 14. Corn log-price with filtered trend for different signal-to-noise ratios.
Generally, the choice of the signal-to-noise ratio is rather arbitrary and is up to the model builder. As we need to simulate the commodity price distributions long time (5 years) into the future, this calls for a relatively small signal-to-noise ratio. In this case, we get a really smooth trend, picking up only large price moves in the past few years, but not the new short term recovery. A larger signal-to-noise ratio would also pick up shorter-term trends, but this might not be too realistic for a 5-year horizon. Therefore, now we turn to the simulation results obtained by the moving block bootstrap procedure to see whether this method is more robust than the mean reversion with trend.

Moving block bootstrap

The moving block bootstrap procedure is used to simulate the returns for the same example of CCO. Again, the simulations are carried out 500,000 times for several block lengths: 2, 5, 10, 20, and 30. In order to investigate the robustness of the block bootstrap approach and the effect of the financial crisis, two sets of historical commodity returns are considered: July 1993 - December 2007 (before the financial crisis), and July 1993 - January 2010 (which includes the highly volatile crisis period, characterized by large declines in commodity prices). Again, two types of default probabilities are used, namely: \( PD = P(0 \leq N_T \leq N_0) \) (part of the notional is lost) and \( PD = P(N_T = 0) \) (all notional is lost).

Exhibits 15 and 16 show the simulated probabilities of default (in %), defined as \( PD = P(0 \leq N_T \leq N_0) \) for each tranche and block length. The first table shows these using only the pre-crisis data, and the second table – including the data during the financial crisis. The standard deviations are omitted for the sake of clarity; their values are comparable to those obtained by the parametric method.

<table>
<thead>
<tr>
<th>Block length</th>
<th>BBB</th>
<th>A</th>
<th>AA</th>
<th>AAA</th>
<th>SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>12.0244</td>
<td>2.0656</td>
<td>1.5744</td>
<td>0.0986</td>
<td>0.01</td>
</tr>
<tr>
<td>5</td>
<td>12.0378</td>
<td>2.2066</td>
<td>1.7098</td>
<td>0.1036</td>
<td>0.0082</td>
</tr>
<tr>
<td>10</td>
<td>12.115</td>
<td>2.4124</td>
<td>1.8588</td>
<td>0.1368</td>
<td>0.0092</td>
</tr>
<tr>
<td>20</td>
<td>11.5966</td>
<td>2.1542</td>
<td>1.645</td>
<td>0.1304</td>
<td>0.0136</td>
</tr>
<tr>
<td>30</td>
<td>11.936</td>
<td>2.2006</td>
<td>1.6856</td>
<td>0.137</td>
<td>0.0102</td>
</tr>
</tbody>
</table>

**Exhibit 15.** Simulated probabilities of default for CCO tranches (block bootstrap method); 1993-2007, \( PD = P(0 \leq N_T \leq N_0) \).

<table>
<thead>
<tr>
<th>Block length</th>
<th>BBB</th>
<th>A</th>
<th>AA</th>
<th>AAA</th>
<th>SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>18.7658</td>
<td>4.8086</td>
<td>3.8178</td>
<td>0.6868</td>
<td>0.1804</td>
</tr>
<tr>
<td>5</td>
<td>18.6986</td>
<td>4.9946</td>
<td>4.0094</td>
<td>0.7054</td>
<td>0.1756</td>
</tr>
<tr>
<td>10</td>
<td>18.2934</td>
<td>5.0558</td>
<td>4.0682</td>
<td>0.836</td>
<td>0.2256</td>
</tr>
<tr>
<td>20</td>
<td>17.478</td>
<td>4.892</td>
<td>3.9176</td>
<td>0.9568</td>
<td>0.3042</td>
</tr>
<tr>
<td>30</td>
<td>17.1264</td>
<td>4.764</td>
<td>3.8204</td>
<td>0.9726</td>
<td>0.3316</td>
</tr>
</tbody>
</table>

**Exhibit 16.** Simulated probabilities of default for CCO tranches (block bootstrap method); 1993-2010, \( PD = P(0 \leq N_T \leq N_0) \).

The first observation is that the results are quite robust to the block length – the differences in default probabilities for different block lengths are just fractions of a percent (or 5-10% of actual values). The second observation is that, when sampling from a wider set of historical returns (1993–2010), the block bootstrap provides higher probabilities of default in comparison to those obtained from 1993-2007 data.
This is especially prominent for highly-rated tranches (AAA and Super Senior), as Exhibit 17 clearly shows. These results suggest the inevitable downgrading of the highly rated CCO tranches in the aftermath of the crisis (but perhaps not as dramatically as it happened to many CDO tranches). The third observation is that the default probabilities implied by both the S&P’s and Fitch’s ratings are always lower than the simulated ones. The above historical performance analysis of Goldman Sachs CCO shows that investors would lose part of their notional in 16.8% (BBB), 2.49% (A) and 1.4% (AA) of the cases. Thus, both the simulations and the historical performance analysis suggest that the default probabilities given by S&P and Fitch are too low.

Exhibit 17. Default probabilities \( (PD = P(0 \leq N_T \leq N_0)) \) for all tranches as given by S&P, Fitch and simulated by the block bootstrap \( (PD = P(0 \leq N_T \leq N_0)) \) with block length 5, for 1993-2007 and 1993-2010.

A similar picture emerges if another definition of the default probability is considered \( (PD = P(N_T = 0) \). Exhibits 18 and 19 show the corresponding simulated probabilities of default. Again, when the crisis data are included, the simulated default probabilities become higher, especially for the highly rated tranches, as clearly shown in Exhibit 20.

<table>
<thead>
<tr>
<th>Block length</th>
<th>BBB</th>
<th>A</th>
<th>AA</th>
<th>AAA</th>
<th>SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.2542</td>
<td>1.7344</td>
<td>0.1152</td>
<td>0.0122</td>
<td>0.0024</td>
</tr>
<tr>
<td>5</td>
<td>2.3918</td>
<td>1.8712</td>
<td>0.1212</td>
<td>0.0092</td>
<td>0.0014</td>
</tr>
<tr>
<td>10</td>
<td>2.6142</td>
<td>2.0394</td>
<td>0.1562</td>
<td>0.0116</td>
<td>0.0016</td>
</tr>
<tr>
<td>20</td>
<td>2.3404</td>
<td>1.8124</td>
<td>0.1518</td>
<td>0.016</td>
<td>0.0034</td>
</tr>
<tr>
<td>30</td>
<td>2.3912</td>
<td>1.8378</td>
<td>0.1598</td>
<td>0.0128</td>
<td>0.0022</td>
</tr>
</tbody>
</table>

Exhibit 18. Simulated probabilities of default for CCO tranches (block bootstrap method); 1993-2007, \( PD = P(N_T = 0) \).
Exhibit 19. Simulated probabilities of default for CCO tranches (block bootstrap method); 1993-2010, $PD = P(N_T = 0)$.

Exhibit 20. Default probabilities $PD = P(N_T = 0)$ for all tranches, simulated by the block bootstrap with block length 5, for 1993-2007 and 1993-2010.

Exhibit 21 shows the default probabilities as functions of the block length. Although the results are quite robust to the block length, an interesting effect is still visible: using longer blocks results in higher default probabilities for senior tranches and lower for the other tranches. This can be explained by the fact that simulating with longer blocks enables capturing longer clusters of simultaneous decrease of the prices (during the crisis period). Since the commodities are strongly correlated during extreme market situations and this dependence is well-captured, a higher number of trigger events at maturity is more likely to occur. The choice of block length can depend on the risk aversion of the investor. Since the underlying risk of a CCO is market risk at long horizons, investors should be cautious and safely choose the block length which gives a higher default probability.
Conclusions

We developed a framework for valuing and rating CCOs that avoids adapting CDO valuation techniques to CCOs – a task for which they are fundamentally unsuited. We examined two methods for obtaining CCO tranche ratings: a parametric model of mean reversion with stochastic trend and a non-parametric data-driven algorithm of block bootstrap. Both methods have the advantage of retaining correlations between commodities which are the essential characteristics of the underlying behavior.

The parametric approach appears to be quite sensitive to the choice of model parameters such as the signal-to-noise ratio. We also investigated the sensitivity of the results to the block length in the block bootstrap method, and found that the results are not significantly sensitive to the block length choice, which indicates the robustness of this method.

We have shown that the nonparametric approach performed remarkably well through the 2008-2010 crisis period, which produced a major stress on commodity prices. When taking into account the data from this, by any definition, “extreme event”, the method has provided sensible and expected results (in terms of potential downgrading of senior tranches).

The suggested nonparametric, data-driven approach is clearly better suited to CCOs than the previous methods employed by the rating agencies, due to the abundance of historical data of commodity prices. The suggested framework is applicable to other structured products that traditionally are dealt with analogously to some other products for historical reasons, while there might be plenty of unexploited historical data.
References


Smith (2002), A tutorial on Principal Components Analysis available at:  

www2.standardandpoors.com/spf/pdf/fixedincome/121905_CDO_e3_assumptionsSNP.pdf

www2.standardandpoors.com/spf/pdf/fixedincome/110206_cco_quantSNAP.pdf?vregin=cn&vlang=en

Appendix: Commodity price graphs

![Copper Price Graph](image-url)
Wheat

Aluminium